Real and nominal frictions within the firm: How lumpy investment matters for price adjustment*

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Abstract

Real rigidities are an important feature of modern sticky price models and are policy-relevant because of their welfare consequences, but cannot be structurally identified from time series. I evaluate the plausibility of capital specificity as a source of real rigidities using a two-dimensional generalized (s,S) model calibrated to micro evidence. Capital lumpiness reduces price stickiness as endogenous fluctuations in the marginal cost of output increase willingness to pay menu costs (an extensive effect), but increases price stickiness through complementarities (an intensive effect). The extensive effect warrants higher menu costs to match evidence on price changes, and the effects of complementarities prevail.

JEL Classifications: E12, E22, E31

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1 Introduction

Infrequent changes in the prices of many final goods and the ability of this simple stylized fact to introduce a role for nominal disturbances has spawned a proliferation of general equilibrium implementations of sticky price models over the last two decades. The prevalent use of this mechanism in models for policy analysis and the observation that counterfactual policy considerations necessitate knowledge of structural parameters (Lucas (1976)) has been very influential, particularly as combined with compelling methods for evaluating calibrated models (Prescott (1986)). Although increasingly nuanced in their implementation, a number of important challenges remain, one of which is the disparity between microeconomic and macroeconomic empirical studies of prices.

Inflation is sufficiently persistent and insensitive to marginal cost movements to imply average price durations of at least 5.9 quarters (Gali and Gertler (1999)) with evidence that this estimate is biased in favor of price flexibility (Linde (2005)). Yet the prices of individual products change frequently with high estimates around 3.0 quarters (Steinsson and Nakamura (2008)) and others are much lower (Klenow and Kryvtsov (2008)). The tension between the observation that prices of individual goods change frequently and the duration of prices implied by the persistence of inflation is successfully mitigated through the introduction real rigidities (Kimball (1995), Sbordone (1999), Gali et. al. (2001)). Interpretation of these real rigidities is ambiguous, however, because a number of plausible structural explanations exist, including variable elasticity of demand (Dotsey and King (2005)), heterogeneity in nominal rigidities across sectors (Carvalho (2007)), and merely increasing short-run marginal cost directly (Woodford (1996)). None of these explanations are identified in macroeconometric studies because the time series can only identify the reduced form parameter.[1] Furthermore, this matters a great deal in terms of the conduct of monetary policy because the source of real rigidities has strong implications for model welfare properties (Levin et. al. (2007)).

One of the more popular ways of generating increasing short-run marginal cost is the inclusion of firm-specific capital, and in this paper I use microeconomic evidence from the U.S. in an attempt to identify the role of firm-specific capital in aggregate dynamics. Firm-specific capital plays an important role in a number of important recent studies, including Smets and Wouters (2007), Murchison and Rennisson (2006), Altig et. al. (2005), Woodford (2005), Sveen and Weinke (2007), and many others, including policy models at

[1] Gali (2005) points this out very clearly in a discussion of Altig, et. al. (2005).

many central banks. Its introduction is justified by the widely shared idea that capital is not freely transferable between firms each period, and it is one of the most widely adopted cases in which interesting dynamics obtain from the interaction of nominal and real rigidities (Ball and Romer (1990)). In each of these models, simulated plant-level investment patterns are counterfactual. Monetary models with capital specificity typically have very smooth firm-level investment rates in which depreciated capital is perpetually replaced each period. Some of these studies use convex adjustment costs (Altig et. al. (2005), Woodford (2005)), and others rely on ad hoc decision rules or complete maintenance investment (Sveen and Weinke (2007)). In contrast, while regular capital maintenance makes up about half of the investment activity in the U.S., the majority of manufacturing plants only partially maintain their equipment in a typical year and compensate for obsolescence and maintenance shortcomings with large but infrequent investment episodes (Doms and Dunne (1998)).

I produce and solve a theoretical model with several key features that enable me to more closely match microeconomic evidence as I reexamine the extent to which firm-specific capital might be a means for resolving the dissonance above: (i) prices change infrequently because of menu costs, (ii) large investment episodes occur infrequently because of fixed installation costs; and (iii) capital maintenance allows machine failures to be repaired without installation costs, but is not sufficient to fully compensate for degradation and obsolescence over time. In my calibration, I match key moments from the price panels for the U.S. Consumer Price Index and from the investment panels in the Longitudinal Research Database before evaluating the implications of the model for aggregate dynamics relative to U.S. time series.

Prices in sticky price models respond to changes in the marginal cost of output just as they would in a purely real model; the partial effect varies as small movements in the marginal cost of output cannot justify any change in price, and large changes in the marginal cost of output will justify payment of a finite cost. A sticky price model with firm-specific productivity fluctuations (Golsov and Lucas (2008)) shows exactly this; and the reasoning is somewhat similar in my model, except that changes in the marginal cost of output are endogenous responses to the incentives to spread fixed installation costs over large capital purchases. The periodicity of large investment episodes and efficacy of intermediate maintenance efforts dictate the magnitude of the changes in the marginal cost of output, and I give the relevant parameters special care in my calibration. Large and frequent price changes accompany large and frequent fluctuations in the marginal cost of output, and small and frequent price changes accompany constant firm-specific production factors (Kimball (1995)).

Holding constant the specification of nominal frictions (i.e., the distribution of menu costs), firm specific capital dramatically mitigates the real effects of nominal disturbances in the economy. A simple menu cost model needs surprisingly small menu costs to match the average duration of prices in the U.S., and changes in capital structure (or in any of the relevant firm-specific states which affect the marginal cost of output) don't need to be very large to motivate the firm to change its prices as well. This is the extensive effect of firm-specific capital documented in previous versions of this paper (Johnston (2007)). Other studies have since confirmed this effect in similar environments (Reiter, Sveen, and Weinke (2009)).

The complementarity effect of short run diminishing returns to scale, frequently because a factor of production is temporarily fixed, is much more well known, and has been for some time (Woodford (1996)). Price changes in a positive inflation environment are, on average, positive; and decreasing returns to scale effectively constrains the magnitude of price increases: marginal cost falls after a relative price increase and gives the firm reason to choose a price lower than it otherwise would (Gali et. al. (2001)). This is the intensive effect.

A more realistic capital structure that matches key moments of U.S. evidence on plant level capital investment patterns justifies much larger nominal frictions if one wishes to continue to also match the evidence on the frequency of price changes. While it's interesting in a theoretical sense to learn about the partial effect on pricing behavior of a change in the capital structure while holding other structural parameters (including those of the distribution of menu costs) fixed, it's not at all a fair comparison in terms of its usefulness for model selection, and the question is more nuanced than the previous paragraphs (and previous versions of this paper) would suggest. I ask which of two menu cost models – one with firm-specific capital, and the other without – produce the most desirable aggregate dynamics when each model is calibrated to match microeconomic evidence on pricing and investment behavior as closely as possible.

The intensive effect remains and is the dominant force in determining aggregate dynamics, after frictions on the nominal side of the model (i.e., the distribution of menu costs) are raised to hold the average duration of prices constant at the level found in microeconomic evidence. Output is higher (and inflation lower) in the year following a moderately persistent shock to the growth rate of money. I conclude that capital specificity is an empirically justifiable real rigidity, although it alone is not sufficient to reconcile microeconomic and macroeconomic evidence. The time series properties of the calibrated model I develop make this clear, and there's an important role for the other real rigidities mentioned earlier.

Section 2 describes the model, beginning with the fairly orthodox description of the household, proceeds to describe the problem of the firm, and concludes with equilibrium conditions that close the model. Section 3 describes the calibration procedure. Section 4 presents the results, considering first the aggregate effects of changing the capital structure of a model while holding the nominal side of the model fixed, and concluding with the implications that the capital structure modification for the calibration of the nominal side. Section 4 concludes.

2 Model

In order to learn more about the possible implications of attached factors for price adjustment, I develop a sticky-price model in which the attached factor is capital. The capital stock of each firm depreciates over time concomitantly raising the marginal cost of output as the production unit requires greater and greater quantities of labor to meet its demands. Investment occurs infrequently as the firm balances the higher marginal cost of output against the benefit of spreading a fixed cost of capital installation across a larger number of units of capital.

Price changes are accordingly infrequent because of small menu costs of adjustment. Each production unit balances the value of selecting a different expected markup path with a small idiosyncratic fixed cost associated with the adoption of the new path.

I describe the choice problem of the firms and households, its solution, and the evolution of the joint distribution of prices and capital goods. I then characterize the connection between the preceding elements and the dynamics of economic aggregates.

2.1 Preferences

Households have preferences over consumption and leisure and maximize the present discounted value of lifetime utility. The household lifetime utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t\right) - \chi N_t \right] \tag{1}$$

where C_t and N_t are aggregates constructed from consumption of differentiated products and hours worked at individual firms.

In addition to labor income $W_t N_t$, households own shares in firms and receive profits Z_t earned through the extraction of monopoly rents and the real return on installed capital. After imposition of a CIA constraint, the household intertemporal budget constraint is

$$C_t + I_t + B_t \leq \Pi_t^{-1} (1 + R_{t-1}) B_{t-1} + W_t N_t + Z_t + Q_t I_{t-1}. \tag{2}$$

Euler equations constrain the demand for bonds and the supply of capital investment goods to firms. The direct interpretation of this setup is that households control production of capital; however, it could also be interpreted as one in which a second perfectly competitive sector of firms perform the trasnformation and return the earnings to the household. In the absence of capital frictions at the firm level, this model reduces to a neoclassical model of investment with pricing frictions.

Intertemporal transfer of wealth can occur through purchases B_t of nominal bonds which promise a net nominal return R_t or through use of an investment technology which converts output into capital with a one period delay. After the delayed transformation of output to capital, the homogeneous capital good is sold to firms at the market clearing price Q_{t+1}, which is the gross real rate of return on investment. This allows for a supply curve for investment goods which is upward sloping a single period in advance but is vertical within a period.

The capital good is produced by the household with a lag but is homogeneous and can be installed and used by firms immediately following installation. Capital homogeneity might seem to be at odds with its concomitant specificity, but this is not so; capital becomes firm-specific when it is installed. The story is that all machines are exactly the same and are produced by a competitive industry and then sold to retail production units. It is specific to a firm when it is bolted down at a plant, but is otherwise identical to the machines at other plants.

Modeling capital markets in this way works well for things like automobiles and copy machines, but not for made-to-order capital goods or specialized production machinery. I selected this structure for tractability, but it seems well-motivated and has been used in prior studies to some extent (Abel, Dixit, Eberly and Pindyck (1996)).

Consumers have isoelastic preferences over a continuum of products such that relative price of a product determines its relative demand,

$$y(p_t, M_t) = Y_t p_t^{-\varepsilon} \tag{3}$$

where ε is the absolute elasticity of demand and M_t is a collection of aggregates that includes Y_t. The output aggregate is composed of a basket of these goods (Dixit and Stiglitz (1977)).

2.2 Technology

The problem of the firm is to choose its current price and capital stock given the current states, including its fixed idiosyncratic capital installation costs and menu costs. Economies with heterogeneous agents are notably more intractable, largely because the state variables and policies of other firms impact future prices. Policies depend on the joint distribution of prices and capital stocks and its evolution.

Solution methods for these economies can be broadly classified into those which approximate the true distribution (Krusell and Smith (1998), Khan and Thomas (2007)) and those which construct economies where the true distribution is a tractable (Dotsey, King, and Wolman (1999), Thomas (2002), Khan and Thomas (2007)). I use the second approach. Idiosyncratic states which induce heterogeneity are i.i.d. so the joint distribution is discrete at each point in time.

A few other studies have examined general equilibrium models with more than one dimension of heterogeneity (Golosov and Lucas (2007), Dotsey, King, and Wolman (2009), Khan and Thomas (2008), Midrigan (2008)), although this is the second study (to my knowledge) in which the additional dimension of heterogeneity is endogenous. This tends to generally complicate the analysis, but I make a simplifying assumption based on the infrequent nature of investment spikes relative to price changes: price and capital can be changed at any time (and even coordinated), but price is assumed to change whenever large capital investment projects are undertaken.

Doms and Dunne (1998) analyze the dynamics of capital investment patterns in manufacturing plants in the U.S. using the Annual Survey of Manufacturers (ASM) of the Longitudinal Research Database (LRD). In the 15-year period from 1973 to 1988, over half of the firms chose an investment rate above 37% per annum; and yet 80% of the plants have an investment rate below 10% per annum. Despite fairly regular maintenance investment, the typical production unit experiences a massive surge of investment following several years without large changes. Steinsson and Nakamura (2008) and Klenow and Kryvtsov (2008) examine the panels of prices underlying the Consumer Price Index in the U.S. and both conclude that most retail goods in the U.S. change prices every three quarters, at most.

Simultaneously, theory suggests that prices will be markups over marginal cost, on average, and that changes in the marginal cost of output very often motivate price changes (Golosov and Lucas (2008)). With tractability as my motivation, and both empirical and theoretical justification, I assume retail prices will change whenever the producer pursues large a large investment project. This condenses the joint distribution of prices and capital to a discrete number of points. A firm's capital stock is unique to its vintage, as are labor demand and output; and at various capital stocks there is a discrete density of prices which are unique to their vintage and capital stock. Together these vintages uniquely identify firms up to their idiosyncratic fixed costs.

Although large investment episodes occur infrequently, there are relatively few cases in which very small investment rates are observed as well. In the ASM, only 18.5% of firms have investment rates below 1% per annum. Matching plant-level stylized facts necessitates both convex and non-convex investment, which I implement by allowing maintenance on broken machines in conjunction with larger capital changes to compensate for non-repairable depreciation and technological obsolescence.

Given its state variables, the firm chooses the optimal reset price-capital combination $\left(\hat{p}_t, \hat{k}_t\right)$ and finds its value,

$$V^{PK}\left(M_t\right) \equiv \sup_{\left(\hat{p}_t,\hat{k}_t\right)\in\mathbb{R}^2_+} \left\{ \begin{array}{c} z\left(\hat{p}_t, \hat{k}_t, M_t\right) - Q_t\left(\hat{k}_t - k_t\right) \\ +E_t S_{t,t+1}\left[V^0\left(\frac{\hat{p}_t}{\Pi_{t+1}}, (1-\delta)\,\hat{k}_t\right) - \varrho\psi k_{t+1}\right] \end{array} \right\} \tag{4}$$

as well as the optimal price $\tilde{p}_t$ supposing its capital stock is fixed and finds its value,

$$V^P\left(k_t, M_t\right) = \sup_{\tilde{p}_t\in\mathbb{R}_+} \left\{ z\left(\tilde{p}_t, k_t, M_t\right) + E_t S_{t,t+1}\left[V^{PK}\left(\frac{\tilde{p}_t}{\Pi_{t+1}}, (1-\delta)\,k_t, M_{t+1}\right) - \varrho\psi k_{t+1}\right]\right\} \tag{5}$$

and finally finds the value of complete inaction,

$$V^{NA}\left(p_t, k_t, M_t\right) = z\left(p_t, k_t, M_t\right) + E_t S_{t,t+1}\left[V^0\left(\frac{p_t}{\Pi_{t+1}}, (1-\delta)\,k_t, M_{t+1}\right) - \varrho\psi k_{t+1}\right]. \tag{6}$$

Adjustment occurs when its net benefit outweighs its net cost *and* the opportunity cost of other types of adjustment. Policies (a_P, a_K) have value

$$V(p_t, k_t, \xi, M_t) = \sup_{(a_P, a_K) \in \{0,1\} \times \{0,1\}} \left\{ \begin{array}{c} (1-a_P)\left(V^{NA}(p_t, k_t, M_t)\right) \\ +A_P \left(\begin{array}{c} (1-a_K) V^P(k_t, M_t) - W_t \xi_{P,t} \\ +a_K \left(V^{PK}(p_t, k_t, M_t) - W_t \xi_{K,t}\right) \end{array} \right) \end{array} \right\} \tag{7}$$

where in each of the expressions above,

$$V^0(p_t, k_t, M_t) \equiv \int_{[0,B_p] \times [B_{kl}, B_{kh}]} V\left(p_t, k_t, \left(\xi_{P,t}, \xi_{K,t}\right), M_t\right) F\left(d\xi_{P,t}, d\xi_{K,t}\right) \tag{8}$$

integrates over the future joint distribution of idiosyncratic adjustment costs, and

$$z(p_t, k_t, M_t) \equiv p_t y(p_t, M_t) - W_t n(y(p_t, M_t), k_t, M_t) \tag{9}$$

is the average firm, $M_t \equiv (C_t, \Lambda_t, W_t)$ is a vector of aggregates, and $S_{t,t+n} \equiv \beta^n \frac{\Lambda_{t+n}}{\Lambda_t}$ is the stochastic discount factor.

At the beginning of each period capital breaks down with probability ψ and needs a capital investment that is distributed Uniform over $[0, 2\varrho]$ to work properly again. Each producer then observes its idiosyncratic adjustment costs $\xi_t \equiv \left(\xi_{P,t}, \xi_{K,t}\right)$ and chooses its price and capital optimally and these become effective immediately. Production occurs using the labor required to clear the market at set prices taking its capital as given. This is as in Gourio and Kashyap (2007), except that the repair cost is random in my model.

At the chosen price and capital levels the firm must hire sufficient labor to clear the market for its output good. Production occurs through a Cobb-Douglas production function

$$y(p_t, M_t) = A_t n(p_t, k_t, M_t)^{\nu} k_t^{\gamma} \tag{10}$$

which determines labor demand. Technological progress takes place through geometric growth in A_t over time at rate Θ_a and all dynamics are presented in terms of deviations or percentage deviations from the balanced growth path this induces. Cyclical fluctuations in productivity follow a first-order autoregressive process with coefficient ρ_A.

Money demand comes from a constant velocity CIA constraint, $m_t = P_t Y_t$. Money growth is stationary at a rate consistent with steady state inflation, and cyclical fluctuations in money growth follow a first-order autoregressive process with coefficient ρ_M.

2.3 Solution

I find the policy and value functions of an average firm in terms of its non-idiosyncratic state variables (i.e., state variables other than the fixed cost realizations today) and the aggregate laws of motion, including the law of motion for the price-capital density. First, I find reset targets $(\hat{p}_t, \hat{k}_t)$ and $\tilde{p}_t$ as they depend on aggregate states and on the firms' historical prices and capital stocks. Second, I use the value of these policies in conjunction with the joint density of idiosyncratic adjustment costs to determine the expected adjustment policies (a_P, a_K). Third, I use this information to describe the dynamics of the joint density of prices and capital, its evolution, and the relationship between firm behavior and aggregate dynamics. Dependence of policy functions on state variables is understood to be implicit and is omitted for brevity.

2.3.1 Euler equations

Reset targets $(\hat{p}_t, \hat{k}_t)$ and $\tilde{p}_t$ are determined by a set of Euler equation recursions. The marginal value of a price is determined by the expected stream of marginal profits it provides.

$$\begin{aligned} \frac{\partial V\left(p_t, k_t, M_t\right)}{\partial p_t} &= \frac{\partial z\left(p_t, k_t, M_t\right)}{\partial p_t} \\ &+ E_t \frac{S_{t,t+1}}{\Pi_{t+1}} \left(\left(1 - \alpha^P\left(\cdot\right) - \alpha^{PK}\left(\cdot\right)\right) \frac{\partial V^{NA}\left(p_{t+1}, k_{t+1}, M_{t+1}\right)}{\partial p_{t+1}} \right) \end{aligned} \tag{11}$$

The marginal value of each reset target is set to zero, $0 = \frac{\partial V(\tilde{p}_t, k_t, M_t)}{\partial \tilde{p}_t}$, and second-order conditions assure their local optimality. This nests the case in which the capital stock is altered simultaneously.

The marginal value of capital comes from its expected reduction in future marginal cost of output as higher levels of capital motivate lower future labor demands (Woodford (2005)), and higher capital purchases today marginally lower anticipated future purchases.

These benefits are partially offset by the requirement that installed capital be maintained.

$$\frac{\partial V\left(p_t,k_t,M_t\right)}{\partial k_t}=\frac{\partial z\left(p_t,k_t,M_t\right)}{\partial k_t} \tag{12}$$
$$+\left(1-\delta\right)E_tS_{t,t+1}\begin{pmatrix}\left(1-\alpha^P\left(\cdot\right)-\alpha^{PK}\left(\cdot\right)\right)\frac{\partial V^{NA}\left(p_{t+1},k_{t+1},M_{t+1}\right)}{\partial k_{t+1}}\\+\alpha^P\left(\cdot\right)\frac{\partial V^P\left(k_{t+1},M_{t+1}\right)}{\partial k_{t+1}}+\alpha^{PK}\left(\cdot\right)Q_{t+1}-\psi\varrho\end{pmatrix}$$

Expected marginal value of capital is equated to the unit cost of capital, $Q_t=\frac{\partial V^{PK}\left(M_t\right)}{\partial k_t}$, and the unit cost of capital floats to clear the market because the supply of investment goods is fixed one period in advance.

2.3.2 Adjustment

I integrate over the joint density of idiosyncratic price and capital adjustment costs, finding optimal behavior at each point, to obtain sets of adjustment probabilities in terms of non-idiosyncratic state variables. For the bivariate uniform density of adjustment costs I employ, analytical expressions for both the adjustment probabilities and expected adjustment costs exist. Arguments of some functions are again implicit and are omitted for brevity. Figure 1 provides an illustration of the case in which $B_l=0$, and is especially useful for thinking about the model when adjustment costs are bivariate uniform, as I assume; in this case each adjustment probability is just the ratio of the dark region to the domain (either dark or light).

Large investment projects accompanied by small price changes occur whenever the associated net change in expected value exceeds the idiosyncratic costs and the change is not dominated by a simple price adjustment.

$$\alpha^{PK}\left(p_t,k_t,M_t\right)=\int_{[0,B_p]\times[B_{kl},B_{kh}]}a^{PK}\left(p_t,k_{t,}M_t,\xi_{P,t},\xi_{K,t}\right)F\left(d\xi_{P,t},d\xi_{K,t}\right) \tag{13}$$
$$=\quad\hat{a}^{PK}\left(\frac{V^{PK}\left(M_t\right)-Q_ti\left(k_t\right)-V^{NA}\left(p_t,k_t,M_t\right)}{W_t},\frac{V^{PK}\left(M_t\right)-Q_ti\left(k_t\right)-V^{P}\left(k_t,M_t\right)}{W_t}\right)$$

where

$$\hat{a}^P\left(x_P,x_D\right)=\frac{\left(\max\left(\min\left(x_D,B_h\right),B_l\right)-B_l\right)}{B_h-B_l}\frac{\max\left(\min\left(x_P,B_p\right),0\right)}{B_p} \tag{14}$$

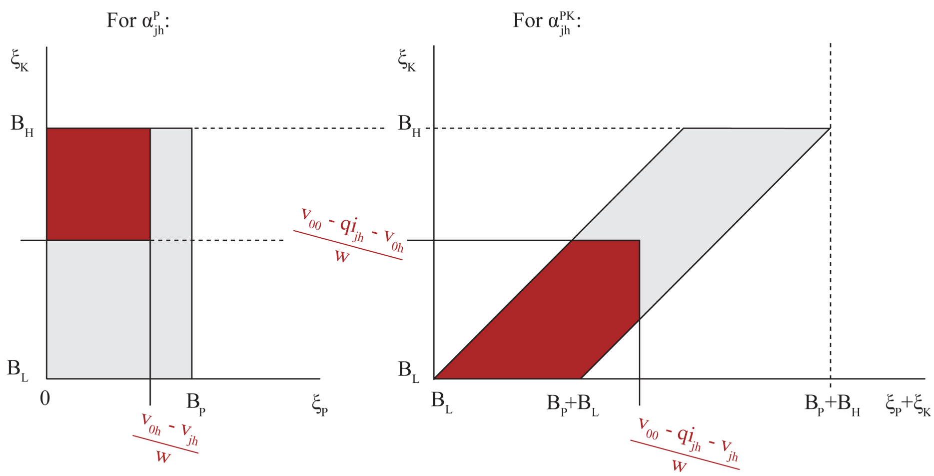

Figure 1: **Figure 1**: Calculation of adjustment proportions. Two conditions are necessary for adjustment: (1) the discrete decision must yield weakly positive net value, and (2) must not be dominated by another discrete decision.

Small price changes absent large investment projects occur whenever the net change in expected value exceeds the idiosyncratic costs and the change is not dominated by a change that incorporates a large capital investment project.

$$\alpha^P(p_t,k_t,M_t) = \int_{[0,B_p]\times[B_{kl},B_{kh}]} a^P(p_t,k_t,M_t,\xi_{P,t},\xi_{K,t})\,F(d\xi_{P,t},d\xi_{K,t}) \tag{15}$$
$$= \hat{a}^P\left(\frac{V^P(k_t,M_t)-V^{NA}(p_t,k_t,M_t)}{W_t}, B_h - \frac{V^{PK}(k_t,M_t)-Q_t i(k_t)-V^P(k_t,M_t)}{W_t}\right)$$

where

$$\hat{a}^{PK}(x_{PK},x_D) = \frac{1}{(B_h-B_l)\,B_p}\left\{\begin{array}{c} \int_{B_l}^{\min(B_h,x_D)} \min(z_2+B_p,x_{PK})\,dz_2 \\ -\frac{1}{2}\left(\min(B_h,x_D)^2-(B_l)^2\right)\end{array}\right\}$$

and the integral in the expression above simplifies to $\frac{1}{2}\left(\min(B_h,x_D)^2-(B_l)^2\right)+B_p(\min(B_h,x_D)-B_l)$ when $\min(B_h,x_D)+B_p \le x_{PK}$, to $x_{PK}(\min(B_h,x_D)-B_l)$ for $x_{PK}\le B_l+B_p$, and is $=\frac{1}{2}\left((x_{PK}-B_p)^2-(B_l)^2\right)+B_p(x_{PK}-B_p-B_l)+x_{PK}(\min(B_h,x_D)-(x_{PK}-B_p))$ otherwise. Expected adjustment costs are computed found analogously, but are omitted from the body for the sake of brevity. Derivations are in one of the technical appendices.

2.3.3 Heterogeneity and its evolution

Adjustment policy likelihoods and the restrictions below provide all of the information necessary to exactly describe the evolution of the discrete density of prices and capital. The end of period density is $\theta\left(k_t, p_t\right)$. Firms either inherit their price and capital states as they evolve from the previous period,

$$\theta\left(p_{t+1}, k_{t+1}\right) = \left(1 - \alpha^{P}\left(p_t, k_t, M_t\right) - \alpha^{PK}\left(p_t, k_t, M_t\right)\right)\theta\left(k_t, p_t\right) \tag{16}$$

or postpone large investment projects and change price,

$$\theta\left(p_{t+1}\left(k_t\right), k_{t+1}\right) = \int_{\mathbb{R}_+} \alpha^{P}\left(p_t, k_t, M_t\right)\theta\left(p_t, dk_t\right) \tag{17}$$

and total firm mass is normalized to unity,

$$1 = \int_{\mathbb{R}_+ \times \mathbb{R}_+} \theta\left(dp_t, dk_t\right). \tag{18}$$

Finally, conditional non-maintenance investment demand is $i\left(k_t\right) = \hat{k}_t - k_t$. Capital otherwise depreciates at rate δ so that $k_{t+1} = \left(1-\delta\right)k_t$. When breakdowns occur they are repaired automatically and a portion of the depreciated capital is automatically replaced at the start of the period. Relative prices stochastically erode with inflation realizations.

2.4 Market clearing

Aggregate resources in the current period are split between consumption and investment and therefore must obey the resource constraint, $C_t + I_t = Y_t$. Investment is an aggregation of demands for capital goods across firms,

$$I_{t-1} = \int_{\mathbb{R}_+ \times \mathbb{R}_+} \left(i\left(k_t, M_t\right)\alpha^{PK}\left(p_t, k_t, M_t\right) + \psi\varrho k_t\right)\theta\left(dp_t, dk_t\right) \tag{19}$$

which includes lumpy investment, $i\left(k_t, M_t\right)\alpha^{PK}\left(k_t, p_t, M_t\right)$ as the product of conditional investment demand and the likelihood of drawing low enough fixed costs, and maintenance on existing capital.

Labor market clearing requires labor supply meet the aggregated labor demand,

$$N_t = \int_{\mathbb{R}_+ \times \mathbb{R}_+} \left(n\left(p_t\right) + \Xi\left(p_t, k_t, M_t\right)\right) \theta\left(dp_t, dk_t\right) \tag{20}$$

where $n_t\left(p_t\right)$ is labor for production and $\Xi\left(p_t, k_t\right)$ is labor for adjusting prices and installing capital conditional on survival.

2.5 Numerical procedure

The economy in the previous section conveniently has a discrete density of prices and capital with known evolution. My interest is in the implications of firm behavior for aggregate dynamics as opposed to firm behavior itself, and for this reason I apply a solution method in the style of Dotsey, King and Wolman (1999) wherein the model equations are linearized around each of the discrete points in the density. The approach is more flexible than standard approaches to linearization, such as linearizing around (p, k), because a differential approximation is taken around each point (p, k) in the discrete density $\theta\left(p, k\right)$. It is potentially more robust than some alternative approaches (Krusell and Smith (1998), Khan and Thomas (2008)) because the evolution of the density $\theta\left(p, k\right)$ and its law of motion are known exactly; other methods necessitate conjectures for the density and its evolution which are difficult to implement, at best, and may be misleading, at worst (den Haan (2008)).

I find the non-stochastic steady state of my model by solving the general equilibrium problem into a sequence of partial equilibrium problems, each of which is solved using standard policy function iteration techniques. In each of the partial equilibrium problems the functional contraction mapping theorems apply and assure convergence. I check market clearing conditions for each partial equilibrium problem and use a Newton-Raphson method to find aggregates at which markets clear from a conjectured vector of initial aggregates. The low dimension of the nonlinear search problem and the contraction mapping theorems which apply to each partial equilibrium problem provide convergence and numerical stability. A dynamic solution to the perturbation of the model equations around the non-stochastic steady state is acquired using standard rational expectations techniques (Sims (2002)).

3 Calibration

A number of parameters are set to match post-war business trends. The discount rate is set to induce an average real interest rate of 4% per year. Technological progress occurs at the geometric rate of 1.6% per year. The autocorrelation coefficient in the process for the log-deviation of aggregate productivity from trend is 0.97. The autocorrelation coefficient on the process for the deviation of the money growth rate from its stationary point is 0.5, a value which allows predictable but not excessively protracted nominal demand movements. I use this simple money supply rule as a diagnostic for judging the behavior of my models, as is common in the New Keynesian literature, and not as a description of actual monetary policy. Returns to scale in production are slightly diminishing at 0.9, a choice consistent with the evidence in Basu and Fernald (1997), and the share of labor in output is 0.64 as in Prescott (1986). The parameter on the disutility of labor in the utility function is selected to generate steady state labor supply of 0.2.

New Keynesian studies have a long tradition of using very high elasticities of demand. I follow microeconometric evidence which indicates that demand elasticities are, at most, one third of the conventional values. For example, Bijmolt, Van Heerde and Pieters (2005) survey more than a thousand microeconometric studies and find an average absolute price elasticity of demand of 2.62, and virtually no microeconometric studies obtain demand elasticity estimates above the value of 12 that is commonly used. I follow empirical evidence and Midrigan (2007) in selecting a demand elasticity of 3.0. Because this value increases the market power of firms and demand is, by definition, less sensitive to price movements in a way that serves to mitigate the impact of firm-specific marginal cost movements on profits. This calibration will, if anything, cause the results I present to be tempered.

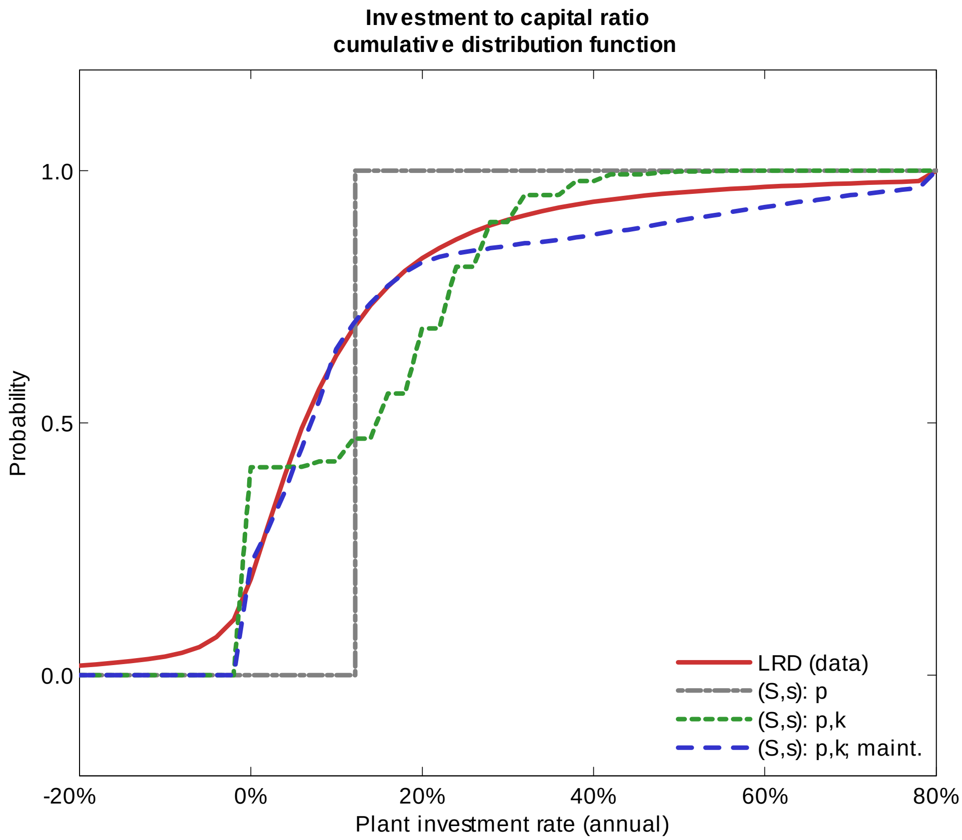

Figure 2: Cumulative distribution functions for the annual investment to capital ratio in three calibrated models and the Longitudinal Research Database as reported in Cooper and Haltiwanger (2006).

Parameters of governing the maintenance, installation, and depreciation of capital are selected to match moments from plant level data in the Annual Survey of Manufacturers (ASM) in the Longitudinal Research Database (LRD) as reported by Cooper and Haltiwanger (2006). First, I match the average investment to capital ratio of 12.2% per year, a number that depends on the depreciation rate, obsolescence rate, the probability of machine failures, and the cost of machine repair in the case of a failure. Higher maintenance investment, either through higher repair costs or lower machine reliability, requires lower depreciation rates to match the observed investment spike rates. Matching this is critical as more effective capital maintenance decreases lumpiness and the depreciation rate net of

Parameters				
Name	*Parameter*	*MLS*	*LS*	*S*
Depreciation rate	δ	4.8% per yr.	12.2% per yr.	
Maximum price adjustment cost	B_p	0.0241	0.0351	0.0037
Minimum capital adjustment cost	B_l	0.0008	0.0004	-
Maximum capital adjustment cost	B_h	0.4015	0.0053	-
Elasticity of demand	ε	3		
Steady-state labor supply	$\bar{n}$	0.2		
Money growth persistence	ρ_M	0.5		
Discount factor	β	0.9902 per qtr., 0.9615 per yr.		
Inflation	π	0.61% per qtr., 2.46% per yr.		
Technology growth rate	Θ_A	0.4% per qtr., 1.6% per yr.		
Share of labor in output		0.64		
Returns to scale		0.9		

Table 1: MLS: the most elaborate model with maintenance investment, lumpiness in investment, and sticky prices. LS: lumpy investment and sticky prices without opportunities for capital maintenance. S: a simple sticky price model with capital.

repairs, an important parameter in this class of models (House (2008)). Second, I minimize the weighted sum of squared differences between the model CDF and the CDF and the cross-sectional plant-level investment rates in the Longitudinal Research Database, where the weights are the proportion of observations found at each investment rate.[2] Figure 2 provides a visual comparison of the cross sectional distribution of plant-level investment rates and compares them with the data.

Finally, I match the median duration of non-sale price changes (without substitutions) in the panels underlying the U.S. Consumer Price Index from 1998 through 2005, as reported in Steinsson and Nakamura (2008); at 7.4 months it is the largest estimate of the elapsed time between price changes found in the U.S. Consumer Price Index over that period. Average inflation is set to 2.46% per year, the average rate in the U.S. between 1998 and 2005, the period included in the most recently analyzed panel of prices from the U.S. Consumer Price Index.

[2] I am grateful to John Haltiwanger and Russell Cooper for providing me with this information.

4 Results

In my discussion of the results, I will focus primarily on the model that best fits the microeconomic evidence: it features both large but infrequent capital investment episodes and regular maintenance of capital breakdowns, in addition to the nominal rigidities introduced through menu costs of price adjustment. The adjustment probabilities implied by the generalized (s,S) policies found earlier are shown in Figure 3, where dark regions correspond to high adjustment probabilities and light regions to low adjustment probabilities. Note how unlikely it is for the firm to undertake a major investment project without waiting several years, a consequence of the lower bound on the fixed cost component associated with major investment undertakings and the effectiveness of capital maintenance activities.

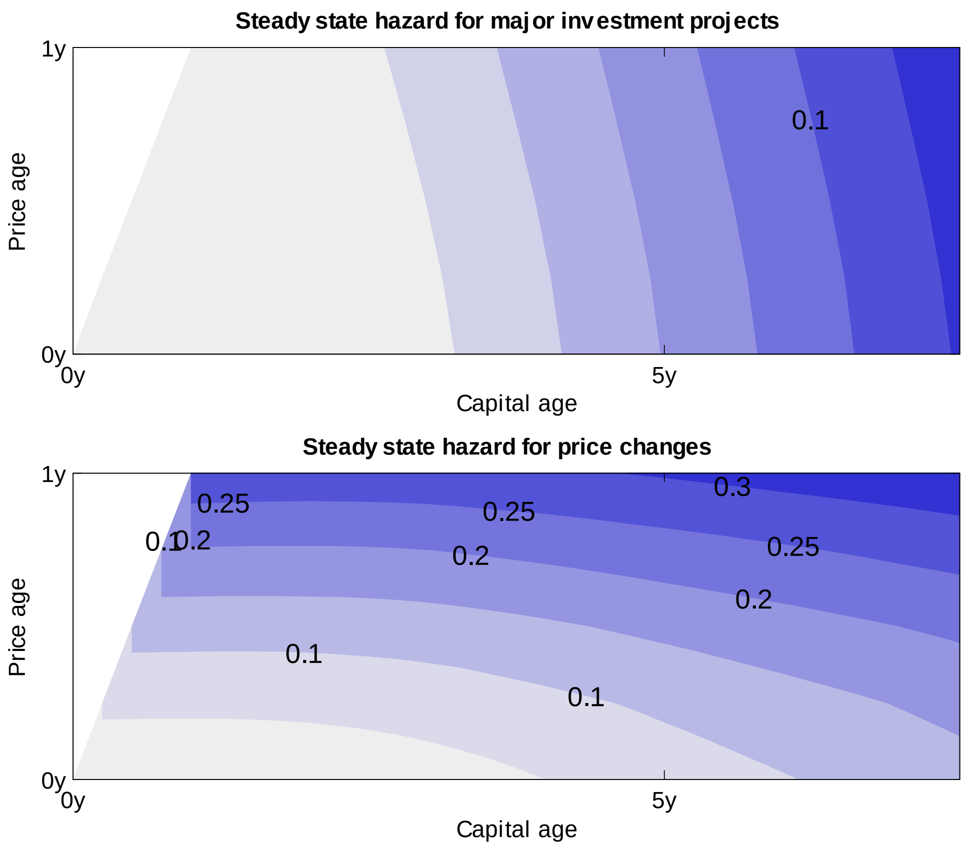

Figure 3: Hazard rates for price changes and large investment projects in steady state for the fully articulated model with capital maintenance.

I begin the discussion of the dynamic implications by holding the nominal frictions constant across all models. Specifically, the results in Figure 4 obtain from a persistent shock to the money supply growth rate when the average duration of product prices is held constant at the Steinsson and Nakamura (2008) levels, and the distribution of menu costs (parameterized by B_p) is adjusted as required. Both of the models with real rigidities have larger and more protracted dynamic responses relative to the simple menu cost model. In the simple (s,S) menu cost model, the upper bound on the support for the distribution of adjustment costs is $B_p = 0.003$, and the average menu cost paid is fairly close to zero. By contrast, the upper bound on the support for the distribution of menu costs is $B_p = 0.045$ in the exclusively lumpy model. The model with capital maintenance has a lower effective depreciation rate (and hence less cross-sectional variation in the marginal cost of output) and requires only $B_p = 0.024$ to match the duration of prices in the U.S. Consumer Price Index.

The dynamic responses confirms, in a more nuanced environment, a well-known propagation mechanism that was first introduced by Woodford (1996), and was subsequently empirically investigated and found to be relevant in studies like Sbordone (1999) and Gali, Gertler, and Lopez-Salido (2001). In a positive inflation environment, the average price change is naturally a price increase, and the size of the increase can be effectively constrained when the marginal cost schedule the firm faces is upward sloping. Price increases lower output, and because of the upward sloping short-run marginal cost schedule, lower marginal cost, providing an incentive for the firm to select a price lower than it otherwise would. Holding the frequency of price changes constant, the aggregate series should behave somewhat more sluggishly when the average change is smaller. While returns to scale in both factors are close to constant (0.9), the inability of the firm to undertake major capital restructuring projects without incurring a fixed cost leaves one factor of production effectively fixed in the short run; the marginal cost of output is determined by the number of workers required to generate an additional unit of output.

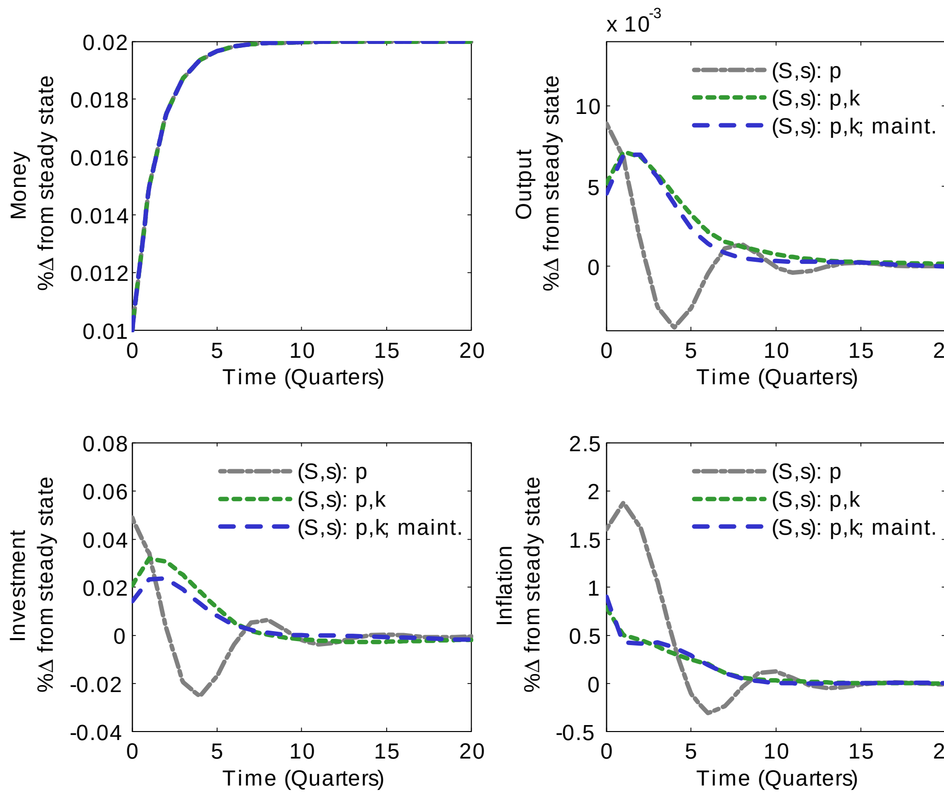

Figure 4: Dynamic responses in output, investment, and inflation, to a persistent shock to the growth rate of the money supply. For this comparison, the support for the distribution of menu costs is varied to hold the average frequency of price changes constant at the level observed in the U.S. Consumer Price Index as reported by Steinsson and Nakamura (2008).

I now examine aggregate dynamics, holding the level of nominal frictions (i.e., the menu cost distribution as parameterized by B_p) constant, as opposed to varying menu costs to hold price durations constant. I fix the distribution at one of the intermediate calibrations found previously and examine the implications of altering the capital structure, holding constant the distribution of nominal frictions. Figure 5 shows that the effect of nominal disturbances on the simple menu cost model are much larger than in the case just considered. Output is almost zero on average over the first year following impact in Figure 4, as discussed earlier, and is large and positive in Figure 5, below.

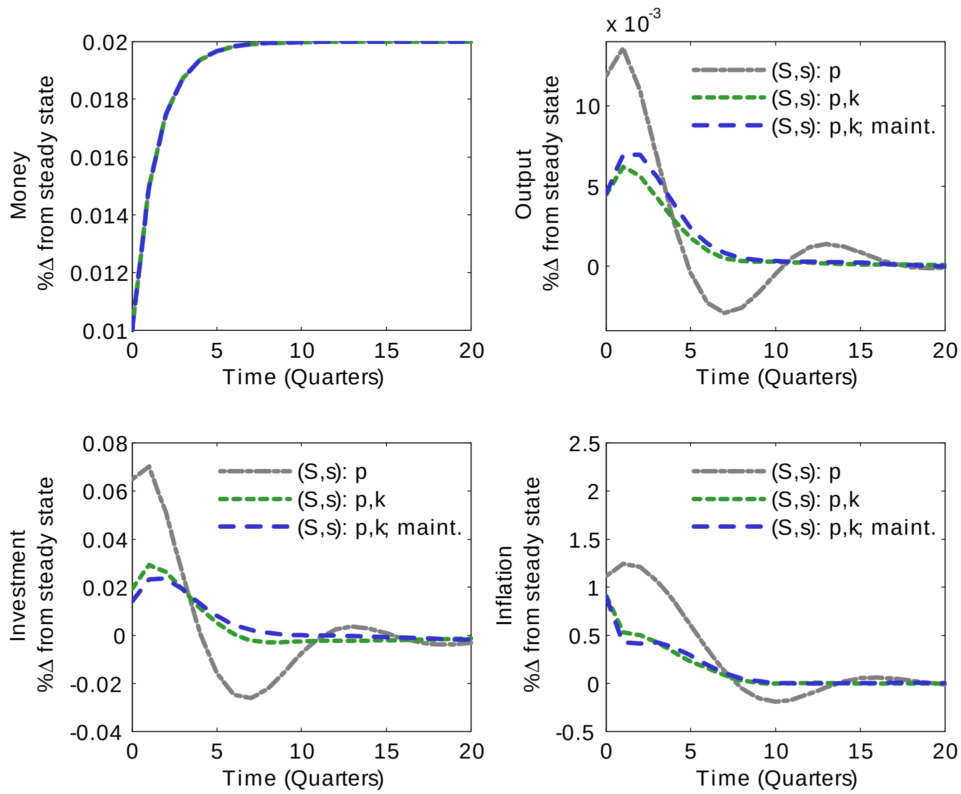

Figure 5: Dynamic responses in output, investment, and inflation, to a persistent shock to the growth rate of the money supply. For this comparison, the distribution of menu costs is fixed across models at the level implied by the calibration of the model with both (S,s) decisions in prices and capital and maintenance of some capital breakdowns.

The cross sectional variation in the marginal cost of output that results from investment lumpiness increases the relative willingness of the firm to pay menu costs of price adjustment, relative to the simple (s,S) sticky price model I consider. Each period, a portion of each firms' effective capital stock decreases, either by obsolescence or depreciation; some of these effects are offset through maintenance, but not all. The marginal cost of output rises with each quarter since the last major capital restructuring project, raising the value to the firm of paying menu costs. Intuitively, one can think about the typical (s,S) story in which proximity to one of the bounds either deterministically or stochastically triggers a discrete decision in which the firm returns the control state in question to its target. As capital structures wear out and become old, workers are increasingly unproductive: this raises the target price that would be chosen under flexibility – a markup over marginal cost – and can be thought of as moving the (s,S) bands and target, making the target more variable and the bounds more frequently encountered.

5 Conclusion

Is firm specific capital, when calibrated to match key properties of U.S. data, capable of increasing the protracted sensitivity of real variables to nominal disturbances? I answer yes, based on results from a two-dimensional generalized (s,S) model which features capital lumpiness and infrequent price changes. While the economic story is somewhat more nuanced than previously – firm-specific capital both motivates price changes, and changes the prices chosen – the idea in the literature that this is a means of producing an upward sloping short run marginal cost schedule (and hence smaller price changes) is robust.

Yet the magnitude to which this mechanism is typically used in the literature is excessive, and the transitive nature of even the most persistent dynamics I present shows that capital specificity can only be one part of the story. There is clearly evidence of sectoral price heterogeneity (Dhyne et. al. (2005)) and theoretical work that supports its efficacy (Carvalho (2007), Steinsson and Nakamura (2009)). So far no there is limited evidence on the variable price elasticity of demand explanation (Dotsey and King (2005)), but the story is intuitively appealing and will hopefully be more closely examined in future research.

References

[1] Abel, Andrew B., Avinash K. Dixit, Janice C. Eberly, Robert S. Pindyck. "Options, the Value of Capital, and Investment," The Quarterly Journal of Economics, Vol. 111, No. 3., pp. 753-777, 1996.

[2] Altig, David, Lawrence Christiano, Martin Eichenbaum and Jesper Linde. "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *NBER Working Paper*, January 2005.

[3] Ball, L. and D. Romer. "Real rigidities and the non-neutrality of money," *Review of Economic Studies*, 57(2), pp. 183-203, 1990.

[4] Basu, Susanto. "Comment on: 'Implications of state-dependent pricing for dynamic macroeconomic modeling'," *Journal of Monetary Economics*, Vol 52, pp. 243-247, 2005.

[5] Basu, Susanto and John G. Fernald. "Returns to Scale in U.S. Production: Estimates and Implications." *Journal of Political Economy*, Vol. 105(2), pp. 249-283, April 1997.

[6] Bijmolt T., Van Heerde H. and R. Pieters. "New empirical generalisations on the determinants of price elasticity," *Journal of Marketing Research*, 42, pp. 141-56, 2005.

[7] Blanchard, Oliver J., and Charles M. Kahn, "The Solution to Linear Difference Models Under Rational Expectations," *Econometrica*, 48: 1305-1311, 1980.

[8] Calvo, Guillermo A.. "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics*, 1983.

[9] Carvalho, Carlos. "Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks," *The BE Journal of Macroeconomics (Frontiers)*, 2007.

[10] Caballero, Ricardo J., Eduardo M. R. A. Engel, John C. Haltiwanger, Michael Woodford, Robert E. Hall. "Plant-Level Adjustment and Aggregate Investment Dynamics," *Brookings Papers on Economic Activity*, Vol. 1995, No. 2, pp. 1-54, 1995.

[11] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 2005.

[12] Cooper, Russell, and John Haltiwanger. "On the Nature of Capital Adjustment Costs," *The Review of Economic Studies*, 2006.

[13] Doms, Mark E. and Timothy Dunne. "Capital Adjustment Patterns in Manufacturing Plants," *Review of Economic Dynamics*, 1(2), 1998.

[14] den Haan, Wouter J. "Assessing the Accuracy of the Aggregate Law of Motion in Models with Heterogeneous Agents," mimeo, University of Amsterdam, 2008.

[15] Dossche, Maarten, Freddy Heylen, and Dirk Van den Poel. "The kinked demand curve and price rigidity: evidence from scanner data," Working Paper No. 99, National Bank of Belgium, October 2006.

[16] Dotsey, Michael and Robert G. King. "Implications of State-Dependent Pricing for Dynamic Macroeconomic Models," Carnegie Rochester Conference Series (edited by Charles I. Plosser and Bennett T. McCallum), *Journal of Monetary Economics*, 52 (1): January 2005, 213-242.

[17] Dotsey, Michael; Robert G. King and Alexander Wolman. "State Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *Quarterly Journal of Economics*, 114(2), May 1999: 655-90.

[18] Dotsey, Michael, Robert G. King, and Alexander Wolman. "Inflation and real activity with firm-level productivity shocks," mimeo, Boston University, 2009.

[19] Gali, Jordi and Mark Gertler. "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 44(2), October 1999.

[20] Gali, Jordi, Mark Gertler, and J. David Lopez-Salido. "European Inflation Dynamics," *NBER Working Paper 8218*, April 2001.

[21] Golosov, Mikhail and Robert E. Lucas, Jr. "Menu costs and Phillips Curves," *The Journal of Political Economy*, 2007.

[22] Gourio, Francois and Anil K. Kashyap. "Investment Spikes: New Facts and a General Equilibrium Exploration," forthcoming, *Journal of Monetary Economics*, 2007.

[23] House, Christopher L. "Fixed Costs and Long-Lived Investments," *NBER Working Paper*, September 2008.

[24] Kimball, Miles S. "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Monety, Credit and Banking*, Vol. 27, No. 4, Nov. 1995.

[25] King, Robert G. and Mark W. Watson. "The Solution of Singular Linear Difference Systems Under Rational Expectations," *International Economic Review*, 39(4), 1998.

[26] Klenow, Peter J. and Oleksiy Kryvtsov. "State-Dependent or Time-Dependent Pricing: Does It Matter for Recent U.S. Inflation?" *NBER Working Paper*, January 2005.

[27] Krusell, Per and Anthony Smith. "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 1998.

[28] Kydland, Finn E., and Edward C. Prescott. "Time to Build an Aggregate Fluctuations," *Econometrica*, Vol. 50, No. 6., Nov., 1982.

[29] Levin, Andrew T., J. David Lopez-Salido, and Tack Yun. "Strategic Complementarities and Optimal Monetary Policy," mimeo, Board of Governors of the Federal Reserve System, 2007.

[30] Levy, Daniel, Mark Bergen, Shantanu Dutta, and Robert Venable, 1997, "The Magnitude of Menu Costs: Direct Evidence from Large US Supermarket Chains," *Quarterly Journal of Economics*, 112(3): 781-826.

[31] Midrigan, Virgiliu. "Menu Costs, Multi-Product Firms, and Aggregate Fluctuations," working paper, Ohio State University, January 2006.

[32] Murchison, Stephen and Andrew Rennisson. "ToTEM: The Bank of Canada's New Quarterly Projection Model," Technical Report 97, Bank of Canada, 2006.

[33] Nakamura, Emi and Jon Steinsson. "Five Facts About Prices: A Reevaluation of Menu Cost Models," forthcoming, *Quarterly Journal of Economics*, 2007.

[34] Reiter, Michael, Tommy Sveen, and Lutz Weinke. "Lumpy investment and state dependent pricing in general equilibrium," *Norges Bank Working Paper*, May 2009.

[35] Sbordone, Argia. "Do Expected Future Marginal Costs Drive Inflation Dynamics?" *Journal of Monetary Economics*, 52(6), September 2005.

[36] Sbordone, Argia. "Prices and Unit Labor Costs: A New Test of Price Stickiness," *Journal of Monetary Economics*, 49(2), March 2002.

[37] Sims, Christopher. "Solving linear rational expectations models," *Computational Economics*, 2002.

[38] Steinsson, Jon, and Emi Nakamura. "Five Facts About Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, 2008.

[39] Steinsson, Jon, and Emi Nakamura. "Monetary Non-Neutrality in a Multi-Sector Menu Cost Model," *Working Paper*, Columbia University, 2009.

[40] Sveen, Tommy and Lutz Weinke. "Lumpy Investment, Sticky Prices, and the Monetary Transmission Mechanis," forthcoming, *Journal of Monetary Economics*, 2007.

[41] Tammo, H.A. Bijmolt, Harold J. Van Heerde, and Rik G.M. Pieters. "New Empirical Generalizations on the Determinants of Price Elasticity," *Journal of Marketing Research*, Vol. XLII, May 2005.

[42] Thomas, Julia. "Is Lumpy Investment Relevant for the Business Cycle?" *Journal of Political Economy*, 110(3), 2002.

[43] Woodford, Michael. "Control of the Public Debt: A Requirement for Price Stability?" *NBER Working Paper 5684*, July 1996.

[44] Woodford, Michael. "Firm-Specific Capital and the New-Keynesian Phillips Curve," *International Journal of Central Banking*, 1(2), 2005.

[45] Yun, Tack. "Nominal Price Rigidy, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics*, vol. 37 (April 1996), pp. 345-70.

6 Appendix A: Adjustment hazards

6.1 Price and capital likelihood

$$
\begin{aligned}
&\hat{a}^{PK}(x_{PK}, x_D) \\
=\ & (B_h - B_l)^{-1}(B_p)^{-1} \int_{-\infty}^{x_D} \int_{-\infty}^{x_{PK}} 1_{[B_l \leq z_2 \leq B_h]} 1_{[z_2 \leq z_1 \leq z_2 + B_p]} dz_1 dz_2 \\
=\ & (B_h - B_l)^{-1}(B_p)^{-1} \int_{B_l}^{\min(B_h, x_D)} \int_{z_2}^{\min(z_2 + B_p, x_{PK})} dz_1 dz_2 \\
=\ & (B_h - B_l)^{-1}(B_p)^{-1} \left\{ \begin{array}{c} \int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK})\, dz_2 \\ -\frac{1}{2}\left(\min(B_h, x_D)^2 - (B_l)^2\right) \end{array} \right\}
\end{aligned}
$$

Obviously, $\hat{a}^{PK}(x_{PK}, x_D) = 0$ if either $\min(B_h, x_D) < B_l$ or $x_{PK} < B_l$. Otherwise, the result can be easily derived in pieces. If $\min(B_h, x_D) + B_p \leq x_{PK}$,

$$
\begin{aligned}
&\int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK})\, dz_2 \\
=\ & \frac{1}{2}\left(\min(B_h, x_D)^2 - (B_l)^2\right) + B_p\left(\min(B_h, x_D) - B_l\right)
\end{aligned}
$$

and if $x_{PK} \leq B_l + B_p$,

$$
\begin{aligned}
&\int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK})\, dz_2 \\
=\ & x_{PK}\left(\min(B_h, x_D) - B_l\right)
\end{aligned}
$$

and for the values of a in between,

$$
\begin{aligned}
&\int_{B_l}^{\min(B_h, x_D)} \min(z_2 + B_p, x_{PK})\, dz_2 = \int_{B_l}^{x_{PK} - B_p} (z_2 + B_p)\, dz_2 + \int_{x_{PK} - B_p}^{\min(B_h, x_D)} x_{PK} dz_2 \\
=\ & \frac{1}{2}\left((x_{PK} - B_p)^2 - (B_l)^2\right) + B_p(x_{PK} - B_p - B_l) + x_{PK}\left(\min(B_h, x_D) - (x_{PK} - B_p)\right)
\end{aligned}
$$

6.2 Price likelihood

$$
\begin{aligned}
& \hat{a}^P(x_P, x_D) \\
= \; & (B_h - B_l)^{-1}(B_p)^{-1}\int_{-\infty}^{x_D}\int_{-\infty}^{x_P} 1_{[B_l \le z_2 \le B_h]} 1_{[0 \le z_1 \le B_p]} dz_1 dz_2 \\
= \; & (B_h - B_l)^{-1}(B_p)^{-1}\int_{B_l}^{\min(B_h, x_D)}\int_0^{\min(B_p, x_P)} dz_1 dz_2 \\
= \; & (B_h - B_l)^{-1}(B_p)^{-1}\left(\max\left(\min\left(x_D, B_h\right), B_l\right) - B_l\right)\max\left(\min\left(x_P, B_p\right), 0\right)
\end{aligned}
$$

6.3 Expected cost

Note: this is not the same $\Xi(\cdot)$ function used in the body of the paper.

$$
\begin{aligned}
& \Xi(x_{PK}, x_P, x_D) \\
= \; & (B_h - B_l)^{-1}(B_p)^{-1}\left\{\begin{array}{c} \int_{-\infty}^{x_D}\int_{-\infty}^{x_{PK}} z_1 1_{[B_l \le z_2 \le B_k]} 1_{[z_2 \le z_1 \le z_2 + B_p]} dz_1 dz_2 \\ + \int_{-\infty}^{x_D}\int_{-\infty}^{x_{PK}} z_1 1_{[B_l \le z_2 \le B_h]} 1_{[0 \le z_1 \le B_p]} dz_1 dz_2 \end{array}\right\} \\
= \; & (B_h - B_l)^{-1}(B_p)^{-1}\left\{\begin{array}{c} \int_{B_l}^{\min(B_h, x_D)}\int_{z_2}^{\min(z_2 + B_p, x_{PK})} z_1 dz_1 dz_2 \\ + \int_{B_l}^{\min(B_h, x_D)}\int_0^{\min(B_p, x_P)} z_1 dz_1 dz_2 \end{array}\right\} \\
= \; & (B_h - B_l)^{-1}(B_p)^{-1}\left\{\begin{array}{c} \int_{B_l}^{\min(B_h, x_D)} \frac{1}{2}\left(\min\left(z_2 + B_p, x_{PK}\right)^2 - z_2^2\right) dz_2 \\ + \frac{1}{2}\min\left(B_p, x_P\right)^2\left(\min\left(B_h, x_D\right) - B_l\right) \end{array}\right\}
\end{aligned}
$$

Because $\hat{a}^{PK}(x_{PK}, x_D) = 0$ if either $\min(B_h, x_D) < B_l$ or $x_{PK} < B_l$, it is also the case that the first component in the expression above is zero under the same conditions. Otherwise, the result can be easily derived in pieces. If $\min(B_h, x_D) + B_p \le x_{PK}$,

$$
\begin{aligned}
& \int_{B_l}^{\min(B_h, x_D)} \frac{1}{2}\left(\left(z_2 + B_p\right)^2 - z_2^2\right) dz_2 \\
= \; & \int_{B_l}^{\min(B_h, x_D)} \frac{1}{2}\left(2 z_2 B_p + B_p^2\right) dz_2 \\
= \; & \frac{1}{2}\left(B_p\left(\min\left(B_h, x_D\right)^2 - B_l^2\right) + B_p^2\left(\min\left(B_h, x_D\right) - B_l\right)\right)
\end{aligned}
$$

and if $x_{PK} \le B_l + B_p$,

$$
\begin{aligned}
& \int_{B_l}^{\min(B_h,x_D)} \frac{1}{2}\left(x_{PK}^2 - z_2^2\right) dz_2 \\
= \; & \frac{1}{2}\left(x_{PK}^2 \left(\min\left(B_h, x_D\right) - B_l\right)\right) - \frac{1}{6}\left(\min\left(B_h, x_D\right)^3 - B_l^3\right)
\end{aligned}
$$

and for the values of a in between,

$$
\begin{aligned}
& \int_{B_l}^{\min(B_h,x_D)} \frac{1}{2}\left(\min\left(z_2 + B_p, x_{PK}\right)^2 - z_2^2\right) \\
= \; & \int_{B_l}^{x_{PK}-B_p} \frac{1}{2}\left(2z_2B_p + B_p^2\right) dz_2 + \int_{x_{PK}-B_p}^{\min(B_h,x_D)} \frac{1}{2}\left(x_{PK}^2 - z_2^2\right) dz_2 \\
= \; & \frac{1}{2}\left(B_p\left(\left(x_{PK} - B_p\right)^2 - B_l^2\right) + B_p^2\left(x_{PK} - B_p - B_l\right)\right) \\
& + \frac{1}{2}\left(x_{PK}^2\left(\min\left(B_h, x_D\right) - \left(x_{PK} - B_p\right)\right)\right) - \frac{1}{6}\left(\min\left(B_h, x_D\right)^3 - \left(x_{PK} - B_p\right)^3\right)
\end{aligned}
$$

7 Appendix B: Steady state

Variables in this section are the stationary equivalents to the balanced growth path variables of the body of the paper.

7.1 Introduction

I use an algorithm that transforms the general equilibrium steady state problem into a sequence of partial equilibrium problems. For a vector of aggregate prices M, I solve the functional fixed point program that is the problem of the firm. Firm policies imply a distribution $\theta(p,k)$ from which I am able to evaluate market clearing conditions. I search over M to find a value satisfying both the implied market clearing conditions $g(M)=0$.

7.2 Market clearing conditions

I solve for $M \equiv \begin{bmatrix} Y & \Lambda & W \end{bmatrix}$ to satisfy $g(M)=0$ for

$$g(M) \equiv \begin{bmatrix} N - \int_{\mathbb{R}_+ \times \mathbb{R}_+} (n(p) + \Xi(p,k))\, \theta(dp,dk) \\ Y - \int_{\mathbb{R}_+ \times \mathbb{R}_+} i(k)\, \alpha^{PK}(p,k)\, \theta(dp,dk) - \Lambda^{-1} \\ Y - \int_{\mathbb{R}_+ \times \mathbb{R}_+} p y(p)\, \theta(dp,dk) \end{bmatrix}$$

7.3 Price reset targets

Given a capital stock, a new price $\tilde{p}$ is independent of its cost of implementation, and satisfies

$$\frac{\partial V(\tilde{p},k,M)}{\partial \tilde{p}} = 0$$

for

$$\frac{\partial V(p,k,M)}{\partial p} = \frac{\partial Z(p,k,M)}{\partial p} + S\left[\Pi^{-1}\left(1 - \alpha^{P}\left(p',k',M'\right) - \alpha^{PK}\left(p',k',M'\right)\right)\frac{\partial V(p',k',M')}{\partial p'}\right]$$

where conditions $\delta > 0$ and $\pi \neq 0$ guarantee this recursion is finite.

7.4 Capital reset targets

Capital $\hat{k}$, chosen in conjunction with price, equates marginal value to marginal cost

$$Q = \frac{\partial V^{PK}(M)}{\partial \hat{k}}$$

where

$$\begin{aligned} \frac{\partial V(p,k,M)}{\partial k} &= \frac{\partial z(p,k,M)}{\partial k} \\ &+ (1-\delta) S \begin{pmatrix} \left(1-\alpha^{P}(\cdot) - \alpha^{PK}(\cdot)\right) \frac{\partial V^{NA}(p',k',M')}{\partial k'} \\ +\alpha^{P}(\cdot) \frac{\partial V^{P}(k',M)}{\partial k'} + \alpha^{PK}(\cdot) Q' - \psi \varrho \end{pmatrix} \end{aligned}$$

and $\delta > 0$.

7.5 Stationary distribution

The stationary distribution of firms across capital and price vintages is determined by

$$\theta\left(p',k'\right) = \left(1-\alpha^{P}(p,k) - \alpha^{PK}(p,k)\right)\theta(p,k)$$

$$1 = \int_{\mathbb{R}_{+}\times\mathbb{R}_{+}} \theta(dp,dk)$$

$$\theta\left(\tilde{p}\left(k'\right),k'\right) = \int_{\mathbb{R}_{+}} \alpha^{P}(p,k,M)\,\theta(dp,k)$$

which is linear in θ over the known (p,k) grid implied by

$$p' = \Pi^{-1} p$$

$$k' = (1-\delta)k$$

and the targets determined previously.

7.6 Adjustment policies

See Appendix A.

8 Appendix C: Model equations

8.1 Growth rates

Variables in this section are *not* identical to those in the body of the paper because they are detrended. Technological progress in the steady state grows at rate Θ_A where the production function is $Y = \int y(j)\, dj = A \int n(j)^v\, k(j)^\gamma\, dj$ so that both Y and $y(j)$ grow at rate Θ_Y. The growth rate of output must satisfy $\Theta_Y = \Theta_A (\Theta_K)^\gamma$. From the resource constraints $C + I = Y$ and $I = \int [k_t(j) - (1-\delta) k_{t-1}(j)]\, dj$, it must be the case that C, I, Y, K and $k(j)$ must grow at the same rate Θ_Y. It must also therefore be the case that $\Theta_Y = (\Theta_A)^{\frac{1}{1-\gamma}}$. From the first order household condition $C^{-1} = \Lambda$ we know $\Theta_\Lambda = (\Theta_C)^{-1}$.

8.2 Household

The representative consumer maximizes the expected present discounted value of lifetime utility, where preferences are separable in time, consumption and disutility of labor.

$$\sup_{\{C_t, N_t, B_{t+1}, I_t\}_{t=0}^{\infty}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \chi N_t \right] \right\} \tag{21}$$

subject to

$$C_t + I_t + B_{t+1} \leq \Pi_t^{-1} (1 + R_{t-1}) B_t + W_t N_t + Z_t + Q_t I_{t-1} \tag{22}$$

where

$$C_t = \left[\sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} (c_{jht})^{\frac{(\varepsilon-1)}{\varepsilon}} \right]^{\frac{\varepsilon}{(\varepsilon-1)}} \tag{23}$$

$$Z_t = \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} (Y_t x_{jht} p_{jht} - W_t n_{jht}) \tag{24}$$

$$N_t = \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} n_{jht} + \sum_{h=1}^{H} \sum_{j=1}^{\min(h,J)} \theta_{jht} \Xi_{jht} \tag{25}$$

and B_t represents real consumer bond holdings, R_t is the nominal interest rate and Z_t contains profits from the firms which the household owns. The household first-order necessary conditions are included in what follows in addition to other aggregate definitions and constraints.

$$\sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t\right) - \chi N_t\right]$$
$$+\sum_{t=0}^{\infty} \beta^t \left(\Theta_\Lambda\right)^t \Lambda_t \left[\Pi_t^{-1}\left(1+R_{t-1}\right)B_t + W_t N_t + Z_t + Q_t I_{t-1} - C_t - I_t - B_{t+1}\right]$$

$$\begin{aligned} 1 &= C_t \Lambda_t \\ \chi N_t^{\phi} &= W_t \\ \Lambda_t &= \beta \Theta_\Lambda E_t \Pi_{t+1}^{-1}\left(1+R_t\right)\Lambda_{t+1} \\ \Lambda_t &= Q_{t+1}\Theta_\Lambda \beta \Lambda_{t+1} \end{aligned}$$

8.3 Block 1: Aggregate variables and restrictions

1.A: Maginal utility of consumption

$$1 = \Lambda_t C_t$$

1.B: Labor supply

$$W_t \Lambda_t = \chi$$

1.C: Consumption Euler equation

$$\Lambda_t = \beta E_t \Pi_{t+1}^{-1}\left(1+R_t\right)\Theta_\Lambda \Lambda_{t+1}$$

1.D: Aggregate investment

$$I_t^L = \sum_{h=1}^{H}\left(k_{0t} - k_{ht}\right)\sum_{j=1}^{\min(h,J)} \theta_{jht}\alpha_{jht}^{PK} + \psi\varrho \sum_{h=1}^{H} k_{ht} \sum_{j=1}^{\min(h,J)} \theta_{jht}$$

1.E: Aggregate resource constraint

$$C_t + I_t = Y_t$$

1.F: Cash in advance constraint

$$m_t = P_t Y_t$$

1.G: Exogenous monetary rule

$$\Delta \ln m_{t+1} = \rho_2 \Delta \ln m_t + e_{m,t+1}$$

1.H: Net inflation

$$\Pi_t = \frac{P_t}{P_{t-1}}$$

1.I: Lagged price level

$$\Pi P^L_{t+1} = P_t$$

1J: Total labour

$$N_t = \sum_{h=1}^{H} \sum_{j=1}^{\min(h,J)} \theta_{jht} \Xi_{jht} + \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} n_{jht}$$

1.K: Price level

$$1 = \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} p_{jht} x_{jht}$$

1.L: The aggregator constraint

This equation is viewed as implicitly defining ζ_t.

$$1 = \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1} \left[1 + \frac{\varepsilon}{1+\kappa} - \frac{\varepsilon}{1+\kappa} \left(1 + \frac{\kappa}{\varepsilon} - \frac{\kappa}{\varepsilon} x_{jht} \right)^{\frac{1+\kappa}{\kappa}} \right]$$

1.M: Investment arbitrage constraint

Investors purchase capital at price 1 and sell it for Q_{t+1}. As these investors are perfectly competitive and have access to the bond with gross return $(1 + R_t)\, \Pi_{t+1}^{-1}$ it must be the case that the following holds. This equation is viewed as defining Q_{t+1}.

$$\Lambda_t = \beta E_t Q_{t+1} \Theta_\Lambda \Lambda_{t+1}$$

1.N: Lagged investment

$$I_{t+1}^L = I_t$$

1.O: SDF

$$S_{t,t+1} = \beta \Theta_Y \Theta_\Lambda E_t \frac{\Lambda_{t+1}}{\Lambda_t}$$

8.4 Block 2: The evolution of firm-level heterogeneity, and firm technology and demand

2.A: Distribution evolution

The mass of firms in vintage $(j+1, h+1)$ at time $t+1$ is the mass of firms in vintage (j, h) at time t who do not choose to adjust capital or price.

$$\begin{aligned} \theta_{j+1,h+1,t+1} &= \left(1 - \alpha_{jht}^P - \alpha_{jht}^{PK}\right) \theta_{jht} \\ for\ h &= 1, ...H-1;\ j = 1, ... \min(h, J-1) \end{aligned}$$

The mass of firms choosing to adjust price, but not capital, defines all but one of the remaining $\{\theta_{jht}\}$ variables.

$$\begin{aligned} \theta_{1,h+1,t+1} &= \sum_{j=1}^{\min(h,J)} \alpha_{jht}^P \theta_{jht} \\ for\ h &= 1, ...H-1 \end{aligned}$$

2.C: Restriction on firm mass

The total mass of the firms must sum to a constant which, in this case, we pick to be unity without loss of generality.

$$1 = \sum_{h=0}^{H-1} \sum_{j=0}^{\min(h,J-1)} \theta_{j+1,h+1,t+1}$$

2.D: Firm-specific capital evolution

Firms which do not adjust their capital stocks see their capital depreciate at rate δ net of maintenance.

$$\begin{aligned} k_{h+1,t+1} &= \frac{(1-\delta)}{\Theta_K} k_{ht} \\ for\ h &= 0, ...H-1 \end{aligned}$$

2.E: Lagged prices

$$\begin{aligned} p^L_{jh,t+1} &= p_{jht} \\ for\ h &= 0, ...H-2;\ j = 0, ... \min(h, J-2) \end{aligned}$$

2.F: Current relative prices

The marginal value recursions imply the H prices p_{0ht} for $h = 0, ...H-1$. Non-reset prices are restricted by this equation.

$$\begin{aligned} p_{jht} &= \left(\frac{1}{1+\pi_t}\right) p^L_{j-1,h-1,t} \\ for\ h &= 1, ...H-1;\ j = 1, ... \min(h, J-1) \end{aligned}$$

8.5 Block 3: Firm output and demand

3A: Production

Production functions are Cobb-Douglas. This equation implicitly defines labour demand n_{jht} since capital is fixed and firms are required to meet demand given their current relative price.

$$
\begin{aligned}
Y_t x_{jht} &= A_t n_{jht}^{v} k_{ht}^{\gamma} \\
for\ h &= 0,...H-1;\ j=0,...\min(h, J-1)
\end{aligned}
$$

3B: Demand

Demand is as in Dotsey and King (2005) and Dotsey, King and Wolman (2007). The parameter ε is the local demand elasticity when $x=1,\ p=1,\ \zeta=1$ and κ is a shape parameter. If $\kappa=-\varepsilon$ this reduces to the standard Dixit-Stiglitz aggregator $x_{jht}=\left(\frac{p_{jht}}{\zeta_t}\right)^{-\varepsilon}$.

$$
\begin{aligned}
x_{jht} &= \left(1+\frac{\varepsilon}{\kappa}\right)-\frac{\varepsilon}{\kappa}\left(\frac{p_{jht}}{\zeta_t}\right)^{\kappa} \\
for\ h &= 0,...H-1;\ j=0,...\min(h, J-1)
\end{aligned}
$$

8.5.1 Block 4: Value function recursions

$$
\begin{aligned}
v_{jht} &= Y_t x_{jht} p_{jht} - W_t n_{jht} \\
&\quad + E_t S_{t,t+1}\left[\begin{array}{c}
\left(1-\alpha_{j+1,h+1,t+1}^{P}-\alpha_{j+1,h+1,t+1}^{PK}\right) v_{j+1,h+1,t+1} \\
+\alpha_{j+1,h+1,t+1}^{PK}\left(v_{00,t+1}-Q_{t+1}\left(k_{0,t+1}-k_{H,t+1}\right)\right) \\
+\alpha_{j+1,h+1,t+1}^{P} v_{0,h+1,t+1}-W_{t+1}\Xi_{j+1,h+1,t+1}-\psi\varrho k_{t+1}
\end{array}\right] \\
for\ h &= 0,...H-2;\ j=0,...\min(h, J-2)
\end{aligned}
$$

$$
\begin{aligned}
v_{J-1,h,t} &= Y_t x_{J-1,h,t} p_{J-1,h,t} - W_t n_{J-1,h,t} \\
&\quad + E_t S_{t,t+1}\left[\begin{array}{c}
\alpha_{J,h+1,t+1}^{PK}\left(v_{00,t+1}-Q_{t+1}\left(k_{0,t+1}-k_{h,t+1}\right)\right) \\
+\alpha_{J,h+1,t+1}^{P} v_{0,h+1,t+1}-W_{t+1}\Xi_{J,h+1,t+1}-\psi\varrho k_{t+1}
\end{array}\right] \\
for\ h &= J-1,...H-2
\end{aligned}
$$

$$\begin{aligned} v_{j,H-1,t} &= (Y_t x_{j,H-1,t} p_{j,H-1,t} - W_t n_{j,H-1,t}) \\ &\quad + E_t S_{t,t+1} \left[\begin{array}{c} v_{00,t+1} - Q_{t+1}(k_{0,t+1} - k_{H,t+1}) \\ -W_{t+1}\Xi_{j+1,H,t+1} - \psi\varrho k_{t+1} \end{array} \right] \\ for\ j &= 0, \ldots \min(h, J-1) \end{aligned}$$

8.5.2 Block 5: Marginal value recursions

Marginal values with respect to capital are given by

$$\begin{aligned} \frac{\partial v_{jht}}{\partial k_{ht}} &= W_t \left(\frac{\gamma}{\nu}\right) \frac{n_{jht}}{k_{ht}} + \left(\frac{1-\delta}{\Theta_K}\right) E_t S_{t,t+1} \left[\begin{array}{c} \left(1 - \alpha^P_{j+1,h+1,t+1} - \alpha^{PK}_{j+1,h+1,t+1}\right) \frac{\partial v_{j+1,h+1,t+1}}{\partial k_{h+1,t+1}} \\ +\alpha^{PK}_{j+1,h+1,t+1} Q_{t+1} + \alpha^P_{h+1,t+1} \frac{\partial v_{0,h+1,t+1}}{\partial k_{h+1,t+1}} - \psi\varrho \end{array} \right] \\ for\ h &= 1, \ldots H-2;\ j = 0, \ldots \min(h, J-1) \end{aligned}$$

$$\begin{aligned} \frac{\partial v_{J-1,h,t}}{\partial k_{h,t}} &= W_t \left(\frac{\gamma}{\nu}\right) \frac{n_{J-1,h,t}}{k_{h,t}} + \left(\frac{1-\delta}{\Theta_K}\right) E_t S_{t,t+1} \left[\begin{array}{c} \alpha^{PK}_{J,h+1,t+1} Q_{t+1} \\ +\alpha^P_{J,h+1,t+1} \frac{\partial v_{0,h+1,t+1}}{\partial k_{h+1,t+1}} - \psi\varrho \end{array} \right] \\ for\ h &= J-1, \ldots H-2 \end{aligned}$$

$$\begin{aligned} \frac{\partial v_{j,H-1,t}}{\partial k_{H-1,t}} &= W_t \left(\frac{\gamma}{\nu}\right) \frac{n_{j,H-1,t}}{k_{H-1,t}} + \left(\frac{1-\delta}{\Theta_K}\right) E_t S_{t,t+1} \left[Q_{t+1} - \psi\varrho\right] \\ for\ j &= 0, \ldots J-1 \end{aligned}$$

$$Q_t = W_t \left(\frac{\gamma}{\nu}\right) \frac{n_{00t}}{k_{0t}} + \left(\frac{1-\delta}{\Theta_K}\right) E_t S_{t,t+1} \left[\begin{array}{c} \left(1 - \alpha^P_{11,t+1} - \alpha^{PK}_{11,t+1}\right) \frac{\partial v_{11,t+1}}{\partial k_{1,t+1}} \\ \alpha^{PK}_{11,t+1} Q_{t+1} + \alpha^P_{11,t+1} \frac{\partial v_{11,t+1}}{\partial k_{1,t+1}} - \psi\varrho \end{array} \right]$$

Marginal values with respect to price are given by

$$
\begin{aligned}
\frac{\partial v_{jht}}{\partial p_{jht}} &= Y_t x_{jht} - \left[Y_t p_{jht} - W_t \left(\frac{1}{\nu} \left(\frac{n_{jht}}{x_{jht}} \right) \right) \right] \left(\varepsilon \left(\zeta_t \right)^{-\kappa} \left(p_{jht} \right)^{\kappa - 1} \right) \\
& + E_t \Pi_{t+1}^{-1} S_{t,t+1} \left[\left(1 - \alpha_{j+1,h+1,t+1}^{P} - \alpha_{j+1,h+1,t+1}^{PK} \right) \frac{\partial v_{j+1,h+1,t+1}}{\partial p_{j+1,h+1,t+1}} \right] \\
for\ h &= 0, ...H-2;\ j = 0, ... \min \left(h, J-2 \right)
\end{aligned}
$$

$$
\begin{aligned}
0 &= Y_t x_{0ht} - \left[Y_t p_{0ht} - W_t \left(\frac{1}{\nu} \left(\frac{n_{0ht}}{x_{0ht}} \right) \right) \right] \left(\varepsilon \left(\zeta_t \right)^{-\kappa} \left(p_{0ht} \right)^{\kappa - 1} \right) \\
& + E_t \Pi_{t+1}^{-1} S_{t,t+1} \left[\left(1 - \alpha_{11,t+1}^{PK} - \alpha_{11,t+1}^{P} \right) \frac{\partial v_{1,h+1,t+1}}{\partial p_{1,h+1,t+1}} \right] \\
for\ h &= 0, ...H-1
\end{aligned}
$$

5G: Price marginal values (terminal H and terminal J vintages) Marginal values with respect to price for terminal H and terminal J firms are

$$
\begin{aligned}
\frac{\partial v_{jht}}{\partial p_{jht}} &= Y_t x_{jht} - \left[Y_t p_{jht} - W_t \left(\frac{1}{\nu} \left(\frac{n_{jht}}{x_{jht}} \right) \right) \right] \left(\varepsilon \left(\zeta_t \right)^{-\kappa} \left(p_{jht} \right)^{\kappa - 1} \right) \\
for\ j &= J-1;\ h = J, ...H-2; \\
and\ for\ h &= H-1;\ j = 1, ...J-1.
\end{aligned}
$$

8.6 Block 6: Adjustment policies and costs

See Appendix A.